Grade 4: Numerical Patterning

4.PR.1, 4.PR.2, and 4.PR.3 (combined outcomes)	
.1 Identify and describe patterns found in tables and charts, including a multiplication chart	1. Identify and describe a variety of patterns in a multiplication chart. 2. Determine the missing element(s) in a table or chart. 3. Identify error(s) in a table or chart. 4. Describe the pattern found in a table or chart.
.2 Reproduce a pattern shown in a table or chart using concrete materials.	5. Create a concrete representation of a displayed in a table or chart. 6. Explain why the same relationship exists between the pattern in a table and its concrete representation.
.3 Represent and describe patterns and relationships using charts and tables to solve problems.	7. Extend patterns found in a table or chart to solve a problem. 8. Translate the information provided in a problem into a table or chart. 9. Identify and extend the patterns in a table or chart to solve a problem.

Clarification of the outcome:

- The three outcomes are strongly inter-related. It makes good pedagogical sense to combine them into one lesson. For example, it would be silly to have students determine a missing element (AI #2) with also having them identify the pattern (AI #9) and extend the number sequence (AI #7 and AI #9).
- The outcomes concern numerical relationships. It <u>does not</u> concern repeating core patterns such as AB AB . . . The outcome also concerns a special numerical relationship. Two ways to describe it are: (1) the relationship between input and output (a mathematical way: x and y stuff in later grades) and (2) the relationship between an independent and a dependent variable (a science way). Students identify input/output relationships that involve only one operation (for example, multiplication). However, a table can involve situations that go beyond this. Students would only consider the relationship between outputs for these situations.

Required close-to-at-hand prior knowledge:

- Arithmetic skills (addition, subtraction, multiplication)
- Familiarity with simple numerical patterns (such as those in skip counting)

SET SCENE stage

Ask students if any of them ever do number puzzles or have come across a situation where entry into something involves figuring out a number pattern (e.g.: an entrance requirement to a club can involve being able to determine what comes next in a sequence such as 2, 9, 20, 35, and so on.). Suggest to students that what they are going to learn concerns getting better at figuring out number sequences.

The problem task to present to students:

Provide students with about four number sequences to extend. Ask them to describe the pattern they used to extend each sequence. See below for examples.

1, 7, 13, 19, ___, ___ (pattern is: add 6)

19, 16, 13, 10, ___, ___ (pattern is: subtract 3)

2, 4, 6, 10, 16, 26, ___, ___ (pattern is: add the two numbers before to obtain the next number)

4, 8, 12, 16, ___, ___ (pattern is: skip count by 4 or add 4)

Comments:

The patterns in the sequences should only involve one operation. The task is a warm-up to the lesson. The task also revisits prior knowledge from grade 3 (increasing and decreasing patterns).

DEVELOP stage

Activity 1: Addresses achievement indicators 2, 4, 7, 8, and 9.

- ✦ Have students present solutions to the SET SCENE task. Accept all valid pattern descriptions. Ensure different ways of describing the patterns are discussed.

	Position Number	Sequence Number
✦ Select a sequence that involves addition (e.g. 1, 7, 13, 19 . . .) and show students how to change the sequence into table form (see below). Discuss how the position number connects to the sequence number. Ask students to complete the table (extend the pattern). Ask students to describe a pattern in the table. Ask them to complete (extend) the table. Ask for and discuss results.	1	1
	2	7
	3	13
	4	19
	5	25
	6	
	7	

	Position Number	Sequence Number
✦ Present a simple table (see example) having a subtraction pattern. Ask students to complete the table (extend the pattern) and to describe a pattern in the table. Ask for and discuss results.	1	83
	2	80
	3	77
	4	74
	5	
	6	
	7	

Activity 2: Addresses achievement indicators 1, 2, 4, 7, 8, and 9.

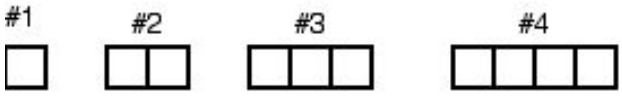
- Provide students with a 9 x 9 multiplication table (1 x 1, 1 x 2, . . . , 9 x 9). Have them identify and describe numerical patterns in the table. Discuss their thinking.

×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

	Position Number	Skip Counting Number
✦ For one of the patterns that involves adding (skip counting) (e.g. row five involves adding 5 each time) ask students to change the skip counting sequence into a table. Ask students to explain how the position number connects to the skip counting number. Ask students to complete the table (extend the pattern). Ask for and discuss results.	1	5
	2	10
	3	15
	4	20
	5	25
	6	
	7	

Activity 3: Addresses achievement indicators 2, 4, 5, 6, 7, 8, and 9.

- Present students with a geometric design series such as the chain of squares shown below. Have students draw/use manipulatives to show the next design, make a table of the data, and figure out how many building blocks are needed to make the 8th design in the series. Discuss their thinking.



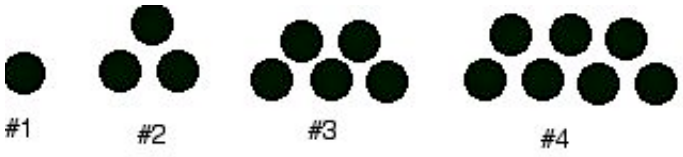
Squares are the basic building blocks for this series.

Design #	Number of squares
1	1
2	2
3	3
4	4

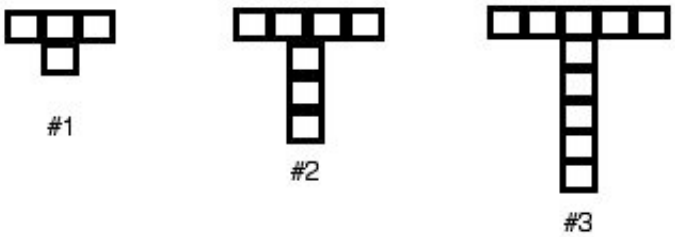
Note:

Students will likely approach this as a number sequence patterning task. In other words, they will look for a number pattern in the building blocks column (the outputs numbers) of the table or they will notice that the design # and the number of squares is always the same. That is fine. Do not develop the INPUT/OUTPUT rule yet. It is developed later in this lesson. For the design series used here, students would likely notice that the number of squares goes up by one each time and use that as a way of figuring out how many squares are needed for the tenth design in the series.

- Repeat the activity with two other design series. See the two examples below.



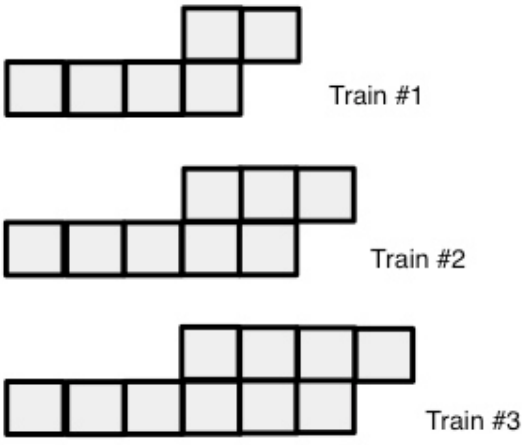
Another series example.



Another series example.

Activity 4: Addresses achievement indicators 4, 5, 6, and 9.

Present students with a table and diagrams that concern trains (see example). Ask students to build train #4 using multi-link cubes (or draw it on grid paper) and to complete table. Ask students to explain why the same relationship exists between the pattern in the table and the train they built/drew. Repeat with two other suitable trains/buildings.



Train #	Number of squares
1	6
2	8
3	10
4	

Activity 5: Addresses achievement indicators 2, 3, 4, 5, and 6.

- Provide students with tables where some numbers in between other numbers are missing (see example). Ask students to describe the pattern and to determine the missing numbers.

cars	people
1	3
2	___
3	9
4	12
5	___
6	___

- Provide students with tables where there are errors in the numbers (see example). Ask students to identify the errors, correct them, and describe the pattern they used to identify the errors.

cars	people
1	4
2	___
3	12
4	15
5	20
6	24
7	29

Activity 6: Addresses achievement indicators 2, 4, 7, and 9.

Present students with a reality-based table involving one arithmetic operation for the input/output relationship (see sample: the relationship between floors and rooms). Discuss the data.

FLOOR	1	2	3	4	5	6	7
ROOMS	3	6	9	12			

Ask students to extend the table. For the example, they would likely realize that the number of rooms increases by 3 each time (the relationship between outputs). They would NOT be expected to realize that the number of 3 x floors is the number of rooms (the input/output relationship). Ask students how they extended the table. Discuss whether their method (adding 3) would be work if they had to figure out the number of rooms for 30 floors. Discuss if there might be a faster method for figuring out the number of rooms for 30 floors and why a faster method might be important to someone building a tall apartment block.

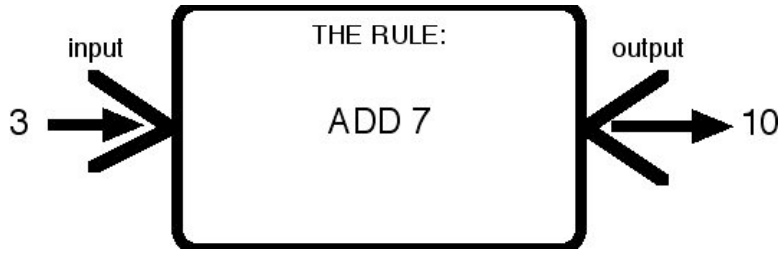
Explain that the faster way involves finding the rule between what comes in (the input) and what goes out (the output). Present the following drawing of an input/output machine.

Present students with input and output in table form that concerns letters of the alphabet (see sample). Enact input coming into the machine and output leaving it, recording the data in the table (e.g. write input A, then write output A). Ask students to complete the table. Ask students what the input/out rule is (one way to describe it is: "make curvy"). Ensure students understand that the rule controls what happens to the input to create the output.

INPUT	OUTPUT
A	A
B	B
C	C
D	D
E	E
F	

Activity 7: Addresses achievement indicators 2, 4, 7, and 9.

Present students with a diagram of an input/output machine (see example). Ask students to make up an input/output rule involving one arithmetic operation (e.g. add 7). Write their rule on a piece of paper and place it inside the machine. Ask students to provide input values and then to calculate the corresponding output values, using the rule. Record these values in a table. Repeat for two other rules.



Activity 8: Addresses achievement indicators 2, 4, 7, and 9. (& practice)

Present a table for which the input numbers are sequential (see example) and the rule involves adding a number (for example, add 4). Ask students to extend the table for three numbers. Ask them to determine the input/output rule and the pattern in the output numbers. Discuss how the input/output pattern/rule (add 4) is not the same as the pattern between consecutive output values (add 1). Repeat for a multiplication input/output rule (for example, times 2).

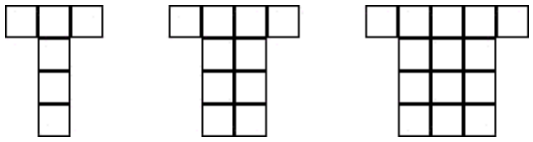
INPUT	1	2	3	4	5	6	7
OUTPUT	5	6	7	8			

Ask pairs of student to create tables where the input/output rule involves addition or subtraction or multiplication (each student of a pair makes two tables). For each pair, ask the students to exchange their tables. The students of each pair complete the given tables by determining and using a input/output rule. Have students check each other's solutions.

Activity 9: Addresses achievement indicators 2, 4, 7, 8, and 9. (& practice)

Provide students with word problems such as in the example.

The letter T can be constructed, using square paving bricks. Each time, the T can be made wider and thicker (see below for the first three T's of the series). How many bricks will you need to build the 40th letter T?



Have students solve the problems by drawing the next diagram, making a table, etc. In the case of the letter T problem here, students would make/draw the next letter T, make a table, identify and use patterns in the table to solve the problem.

Ask if students now feel that they are better at detecting and extending number patterns. Ask and discuss if they now would find it easier to solve number puzzles in such things as games.

Activity 10: Assessment of teaching

Present students with a couple of input/output tables (see example), in other words, a short worksheet. The input/output rule can involve only ONE arithmetic operation (add/subtract/multiply). Ask students to extend the table for three more numbers and to determine the input/output rule AND the pattern between consecutive output values.

INPUT	OUTPUT
3	1
4	2
5	3
6	4
7	
8	
9	
20	

Students would be expected to extend the table (5, 6, 7), figure out the input/output rule (subtract 2), and use it to figure out the output when the input is 20 (20 - 2 = 18).

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

An example of a partially well-designed worksheet follows.

The worksheet contains a sampling of question types. More questions of each type are needed for a well-designed worksheet.

The MAINTAIN stage follows the sample worksheets.

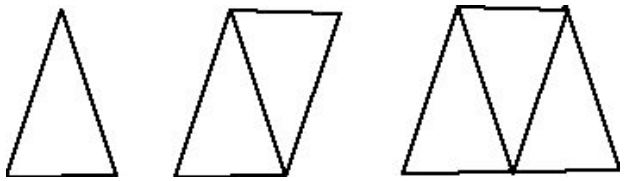
Question 1.

Complete the following table by finding and using a pattern.

Describe the pattern you used.

CARS	PEOPLE
1	4
2	8
3	12
4	16
5	
6	

Question 2.



Draw the next triangle chain.

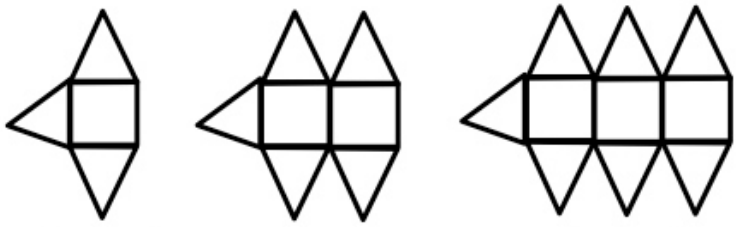
For the first four triangle chains, make a table showing the train # and the number of triangles needed to build it.

Describe two patterns in your table. One of them must be the input/output rule.

Use the input/output rule to figure out how many triangles are needed to build triangle train #40.

Question 3.

How many triangles will be needed for the eighth drawing? Make a table as part of showing your work.



Drawing #1 Drawing #2 Drawing #3

MAINTAIN stage

Mini-task example

From time to time, present students with at-chart and ask them to determine the horizontal pattern (the relationship between input and output when the chart is organized downwards) and the vertical pattern.

Rich-task example

There are 7 flags arranged along a line in race course. Bob runs to each of the 7 flags in a race. The first flag is 200 metres from the start. After that, each flag is 50 metres further from the one before it. How far from the start is the 5th flag? Draw a diagram of the race course, showing distances between flags. Explain how you solved the problem.

Comments.

This is a rich-task because of the complexity of the patterning problem.